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An Analysis of Log Periodic Antenna with Printed Dipoles

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***Abstract*—An analysis of Log Periodic Antenna with Printed Dipoles is presented here. In this analysis, the wave equation for Hertz potential is solved in Cartesian coordinates applying the boundary conditions of a flat strip dipole. Using this model, the input currents to the antenna elements, the current distribution of the antenna elements, and the radiation pattern are computed. The computed results are compared with experimental results.**

I. Introduction

In RECENT years frequency independent antennas [1] have gained significant importance. The Log Periodic Dipole Array (LPDA) is an important type of frequency independent antenna and was invented by Isbell [2] at the University of Illinois in 1958.

Several theories based on the transmission line approach have been put forth for the analysis of LPDA [3], [4], [5]. Wolter [6] derived a theory of Log Periodic Dipole Antenna as a solution of the antenna boundary value problem. He calculated the current distribution on antenna elements by solving the wave equation for Hertz potential in cylindrical coordinates, satisfying the appropriate boundary conditions.

At microwave frequencies, wire dipoles may be bent due to rough handling causing asymmetries in the structure, which, in turn result in back radiation and side lobes [7]. Therefore, it is better to replace the wire antenna by printed dipole which is more rugged and can be easily fabricated.

In this paper, a mathematical model for the analysis of LP array using printed dipole is developed following Wolter's method [6]. However, in this case, the wave equation for Hertz potential is solved in rectangular coordinates satisfying the boundary conditions of a flat strip dipole.

II. Analysis

The antenna consists of N parallel flat strip dipoles. The antenna lies in the x–y plane of the rectangular coordinate system as shown in Fig. 1. The details of the nth element are shown in Fig. 2 and the dimensions of the test array are given in Table I. The elements are fed by a symmetrical transmission line with the characteristic impedance z_0. The two conductors of the transmission line are separated by a dielectric sheet of thickness t. An extra phase shift of 180° is introduced by switching the connection of the adjacent elements.

In the following analysis the dipole elements are assumed infinitely thin and perfectly conducting for the sake of simplicity. If t is infinitesimally small, the two strips of elements can be considered to be at $z=0$. Taking the time variation as $\exp(j\omega t)$ the wave equation for Hertzian vector will reduce to

$$\Delta\pi_n + K_0^2\pi_n = 0 \tag{1}$$

where K_0 is the wavenumber in free space. π_n will have only y component due to the choice of coordinate system. Since each element is symmetrical in x, y, and z about its

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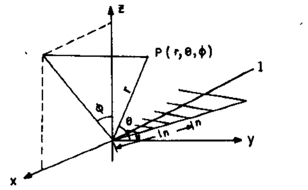

Fig. 1. The antenna in a system of Cartesian and spherical coordinates, $x=r\sin\theta\sin\phi, y=r\cos\theta, z=r\sin\theta\sin\phi$.

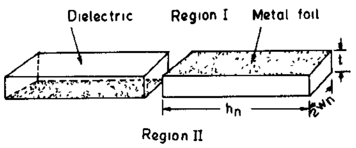

Fig. 2. Details of the nth element.

TABLE I
PARAMETERS OF TEST ANTENNA

$\tau = 0.867$, $\sigma = 0.152$, $L/\lambda_{max} = 0.977$, $Z_0 = 69.106$ ohms, $N = 15$

S.No. of dipole	Height 'h_n' of dipole in cms	Distance 'l_n' from apex in cms	Width '$2w_n$' of dipole in cms
1	9.375	42.857	1.172
2	8.128	37.157	1.016
3	7.047	32.215	0.881
4	6.110	27.931	0.764
5	5.297	24.216	0.662
6	4.593	20.995	0.574
7	3.982	18.203	0.498
8	3.452	15.782	0.432
9	2.993	13.683	0.374
10	2.595	11.863	0.324
11	2.250	10.285	0.281
12	1.951	8.917	0.244
13	1.691	7.731	0.211
14	1.466	6.703	0.183
15	1.271	5.812	0.159

Width of balance feeder w_b = 3.675 mm.

center, then π_n for the given value of k_x and k_y is given by

$$\pi_n = Ce^{-\gamma|z_n|}\cos k_x x \cos k_y y. \tag{2}$$

To make it more general, it should be written in Fourier integral representation

$$\pi_n = \frac{1}{\pi^2}\int_0^\infty\int_0^\infty C_n(k_x, k_y)\cos k_x x_n \cdot \cos k_y y \cdot e^{-\gamma|z_n|}dk_x dk_y. \tag{3}$$

The current on the dipole is assumed to be in y direction only and the surface current density is given by

$$J_y(x, y) = H_x|_{z=0^+} - H_x|_{z=0^-}. \tag{4}$$

The boundary condition (4) leads to

$$\frac{I_n(y)}{2w_n} = \frac{j\omega\epsilon_0}{\pi^2}\int_0^\infty\int_0^\infty 2\gamma C_n(k_x, k_y)\cos k_x x_n \cdot \cos k_y y\, dk_x dk_y \tag{5}$$

where $I_n(y)$, the current on the nth dipole assumed uniformly distributed across the element, can be represented in a Fourier series with only odd cosine terms, because $I_n(h_n)=0$:

$$I_n(y) = \sum_{s=1,3,5\cdots} A_{ns}\cos\left(\frac{s\pi}{2}\frac{y}{h_n}\right). \tag{6}$$

Following Wolter [6], currents on the antenna elements, extended to infinity by current less nonmetallic strips, can be expressed as a Fourier integral and one can obtain the Hertz vector π_n due to the nth element. The Hertz vector for the total radiation field π_t can be obtained by summing up the fields for the individual elements.

The coefficients A_{ns} in (6) can be determined by the use of boundary condition for the electric field on the surface of the metal antenna and the voltage conditions at the feeding points of the dipoles

$$\frac{\partial\pi_t}{\partial y}\Big|_{z=0,x=l_v,y=0^+} - \frac{\partial\pi_t}{\partial y}\Big|_{z=0,x=l_v,y=0^-} = U_v \tag{7}$$

where U_v is the feeding voltage of the element v and

$$E_{yv} = E_y|_{z=0,x=l_v} = \left(k_0^2 + \frac{\partial^2}{\partial y^2}\right)\pi_t|_{z=0,x=l_v} = 0 \tag{8}$$

on the metal part of the dipole v.

From these, one gets the following infinite set of linear equations from which A_{ns} can be calculated:

$$\sum_{s=1,3,5,\cdots}\sum_{n=1}^{N} Z_{mvns}A_{ns} = U_v, \qquad m=1,3,5,\cdots, \qquad v=1,2,\cdots,N \tag{9}$$

where

$$Z_{mvns} = \frac{8jh_nh_v}{\pi^3\omega\epsilon_0}\frac{sm(-1)^{s+m}}{w_n}\int_0^\infty\int_0^\infty \frac{\sin k_x w_n}{k_x\gamma}(k_0^2 - k_y^2) \cdot \cos(k_x x_{nv})\frac{\cos k_y h_n}{s^2 - \left(\frac{2h_n}{\pi}k_y\right)^2} \cdot \frac{\cos k_y h_v}{m^2 - \left(\frac{2h_v}{\pi}k_y\right)^2}dk_x dk_y. \tag{10}$$

The coefficients Z_{mvns} are impedances representing interaction between the elements. The feeding voltage U_{n+1} is known and other feeding voltages U_v can be calculated in terms of I_n (0) and the geometrical antenna parameters based on a transmission line approach [6].

Once A_{ns} coefficients are known, current distribution on the antenna elements are obtained from (6). The Hertzian vector and electric field can also be obtained.

The electric field will be given by

$$E_y = \frac{-j}{8\pi\omega\epsilon_0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sum_{n=1}^{N}\frac{k_0^2-k_y^2}{w_n}\,\frac{\sin k_x w_n}{k_x\gamma}$$

$$\cdot\sum_{s=1,3,5,\cdots}\frac{4h_n}{\pi}A_{ns}\,\frac{s(-1)^{(s-1)/2}}{s^2-\left(\frac{2h_n}{\pi}k_y\right)^2}\cos k_y h_n\cdot\cos k_x x_n$$

$$\cdot\cos k_y y\cdot e^{-\gamma|z_n|}\,dk_x\,dk_y. \qquad (11)$$

In the far field E_y must take the form

$$E_y \sim f(\theta,\phi)\frac{e^{-jk_0 r}}{r}\,. \qquad (12)$$

If the far field approximation is applied, the integral equation (11) can be solved with the help of stationary phase method. In the spherical coordinates (Fig. 2):

$$E = \frac{-K}{\omega}\,\frac{k_0}{\sin\phi}\,\frac{e^{-jk_0 r}}{r}\sum_{n=1}^{N}\sin\left(k_0 w_n\sin\theta\sin\phi\right)$$

$$\cdot\exp(-jk_0 l_n\sin\theta\sin\phi)\,\frac{h_n}{w_n}\cos\left(k_0 h_n\cos\theta\right)$$

$$\sum_{s=1,3,5,\cdots}A_{ns}\frac{s(-1)^{(s-1)/2}}{s^2-\left(\frac{2h_n}{\pi}k_0\cos\theta\right)^2}\hat{i}_\theta \qquad (13)$$

where K is a constant and $\hat{i}_\theta$ is a unit vector in θ direction. From this equation radiation pattern can be computed.

III. Results

An antenna for S-band was designed by Carrel's method [3] by making use of the equivalence between a cylindrical dipole and a flat strip dipole. The antenna was designed for a directive gain of 9 dB and input impedance of 50 Ω. A perspex sheet of 1.5-mm thickness was used as the dielectric ($\epsilon_{\text{eff}}=2.62$). The antenna was fabricated by using a thin copper foil.

In the calculations, the Fourier series was approximated by its three terms in order to save computer time in the existing computer [IBM 6044]. For our test antenna even at the highest frequency considered (3.2GHz) $h_n/\lambda<1$ and convergence of the Fourier series was good. Wolter [6] pointed out that for $h_n/\lambda<1.25$, the five-term approximations are very good. Cheong and King [4], [5] used a three-term theory for accurate determination of antenna parameters for $h_n/\lambda<0.625$.

Fig. 3 shows the computed amplitude of the input currents to the antenna elements. The results are similar to the available results for LP array with cylindrical dipoles [6]. It confirms that at any frequency only a part of the antenna (active region) is in resonance, the amplitude of the input currents to the antenna elements drop off rapidly on both sides of active region and as the frequency is increased, the active region moves towards apex. Fig. 4 shows the computed current distributions on the antenna elements. As expected, the current distribution on a resonant antenna element is the same as on a half wave dipole. The current along the long elements drops as $\cos(\pi y/2h_n)$, and on the short elements there is almost a linear decline of current.

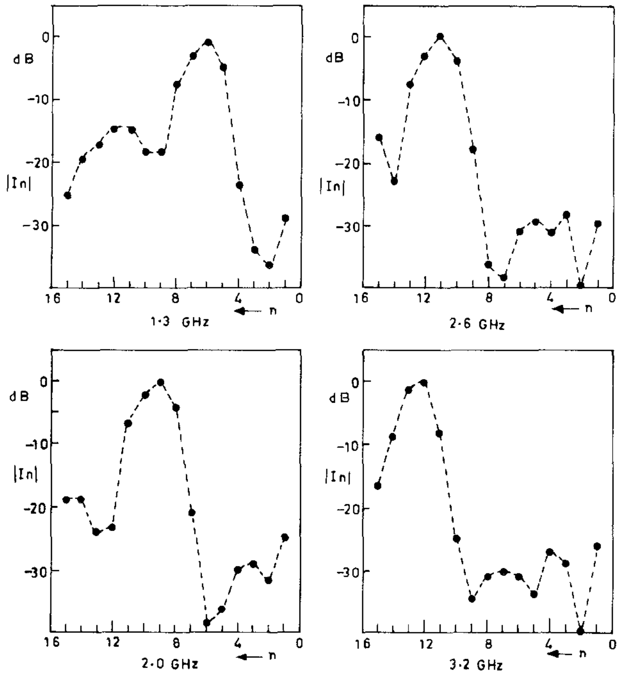

Fig. 3. Computed amplitude of the input currents to the antenna elements (only the indicated points are important).

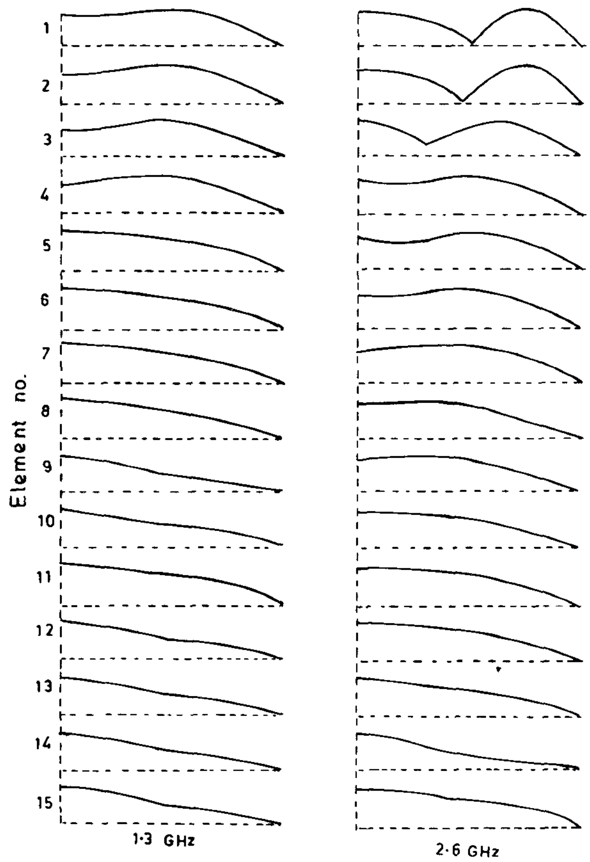

Fig. 4. Computed current distribution on the antenna elements. y/h_n is the abscisssa and $|I_n(y)|/|I_n|_{\max}$ is the ordinate.

Fig. 5 shows the measured radiation patterns along with

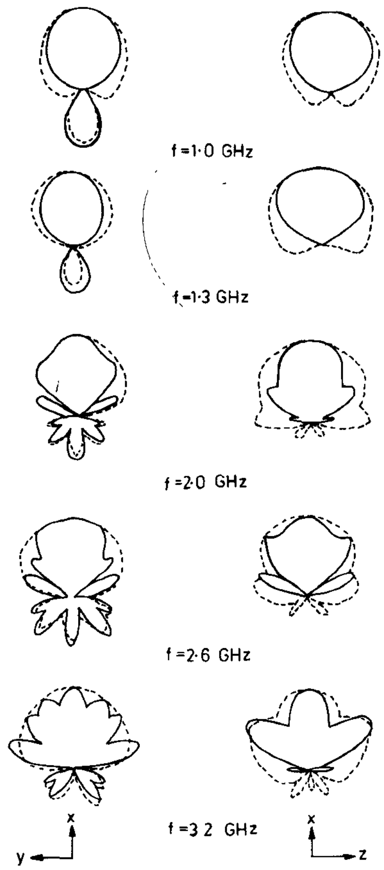

Fig. 5. Computed (—) and measured radiation pattern (- - -).

the computed radiation patterns. The radiation patterns are shown in x–y and x–z plane. It is clear that the beam is always directed towards the apex of the antenna. Frequency independent behavior is observed over the designed range of the antenna. The computed pattern is in reasonably good agreement with the measured radiation pattern. We strongly believe that the discrepancies between the experimental and computed pattern are primarily due to the experimental errors. The experiments were performed on a roof top in free space with the antenna mounted on a manually rotated turntable. Apart from the human error and wind effects, there might have been reflections from surrounding objects. Thus, slight discrepancies in the radiation pattern, e.g., displacement of side lobes etc. in the x–y plane can be explained. The discrepancies in the radiation pattern in the x–z plane are more different in form and the back radiation and side lobes observed in experimental curves may be due to the asymmetries in the fabricated antenna. Furthermore, the Balun was not investigated thoroughly and a locally available design was used.

IV. Conclusion

An analysis of a log periodic array with printed dipole has been done as boundary-value problem by solving the wave equation for Hertz potential in rectangular coordinates applying boundary conditions on a flat strip dipoles. This theory can be extended to other configurations using linear parallel flat strip dipoles.

One limitation of this analysis is that the effect of dielectric is not considered. One way to take this into account is to assume that the antenna is fully immersed in the dielectric, which can be easily done by replacing k_0 by propagation factor in the dielectric and then modify the calculated results by a correction factor [8].

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